Macro Hive: Duration and Convexity in Bond Markets

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Summary

- In this explainer, we cover how to price a bond, how to use duration to predict bond price changes, and how to use convexity to get better estimates of profit and loss.
- The material covers a Macro Hive YouTube video, and Excel examples follow those provided in 'Fixed Income Mathematics' by Robert Zipf.

Understanding the Bond Market

Bond market concepts can be daunting. Many require an understanding of compound interest, present values, discounting, and differential calculus. In this explainer, we go through all the things an investor needs to know to understand the concepts of *duration* and *convexity*. And we will supplement the explainer with a number of examples in Excel.

What Is a Bond?

A bond is simply a tradable loan. It comprises a *principal amount*, equivalently known as the bond's face or par value, which equals the amount borrowed. It also contains periodic 'fixed' interest payments that are usually paid annually (e.g., France or Germany) or semi-annually (e.g., the US) at a predetermined *coupon rate*.

The principal amount is then paid back, alongside the final interest payment, on the date the bond matures, i.e. the *maturity date*. Typically, *short-dated bonds* are those with a maturity of fewer than five years. *Intermediate bonds* or *the belly of the curve* refers to five- to 10-year bonds. And *long-dated bonds* are those with maturities of more than 10 years.

Most bonds are *coupon bonds*, but there are also *zero-coupon bonds*. The best-known zero-coupon bonds are US savings bonds. These pay the principal amount at maturity, alongside all the interest you would have earned.

Trading Bonds

Treasury markets are some of the most liquid markets around the world. As a result, bonds issued by governments are almost always used as the *benchmark bond*. On the global stage, the benchmark is the US, with everything else seen as a spread relative to the US.

The importance of benchmarks is not just limited to measuring a bond's risk or return. Their levels also reflect the market's view on interest rates, inflation, public-sector debt and economic growth. As such, there is a broad spectrum of investors, from short- to long-term institutional investors (e.g., banks and pension funds), and market makers to proprietary traders.

When an investor is said to be trading either bonds, rates or fixed income, they broadly mean the same thing. When an investor is said to be *bullish bonds* or *long bonds*, it means they expect the price of bonds to increase, which is equivalent to the yield going down.

Most investors do not hold a bond to its maturity. For the period that they do hold a bond, they will receive their coupon payments. But they can also make a profit or loss depending on how the price of a bond has changed, just like any stock.

The bond industry describes bonds in a standard form: the issuer name, coupon rate, maturity date or time to maturity, and the offering yield or price. For example, a 5.5% 1y US Treasury issued bond with price 105 would be ‘Treasury 5 1/2s, due 25/03/2023 at price 105’.

Pricing a Bond

The price of a bond is determined by (i) the accrued interest a buyer earns over the maturity of the bond and (ii) the final amount paid on the maturity date. To account for the fact money is worth more today than tomorrow, we need to take the *present value* of these terms. To do so, you just divide today’s amount by the amount of interest you would have accrued if it was in the bank (Table 1).

Table 1: Calculating Bond Price of 10y Bond With 5% Coupon Rate and 1% Yield

M	N	O	P	Q	R	S
78						
79		**Interest**		**Principal**		**Bond**
80	**Years**	**Today's Value**	**Present Value**	**Today's Value**	**Present Value**	**Price**
81	1	5	4.95	0	0	
82	2	5	4.90	0	0	
83	3	5	4.85	0	0	
84	4	5	4.80	0	0	
85	5	5	4.76	0	0	
86	6	5	4.71	0	0	
87	7	5	4.66	0	0	
88	8	5	4.62	0	0	
89	9	5	4.57	0	0	
90	10	5	4.53	100	90.53	
91	**Accrued**		47.36	+	90.53	137.89
92		O81/(1+1%)^N81				
93						

Price-Yield Curve

There is an inverse relationship between the price of bonds and bond yields. If we take the above example, but allow the yield to vary 1-10%, we can trace out the price-yield curve (Chart 1). The higher the yield, the lower the price, i.e., the curve is downward sloping.

Chart 1: Price-Yield Curve

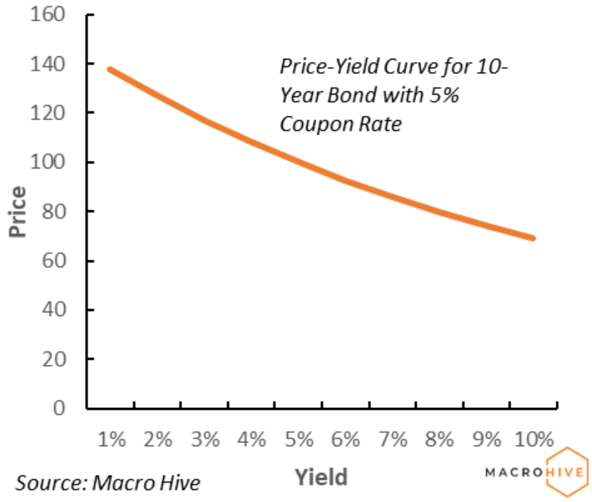


Chart 2: Convexity Increases With Maturity

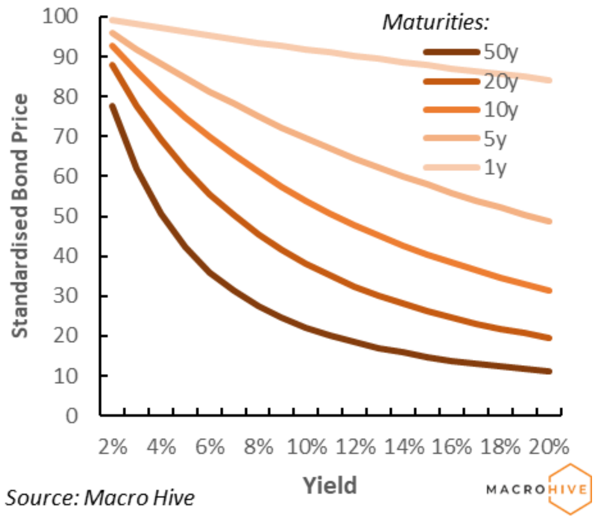


Chart 3: Convexity Decreases With Coupon Rate

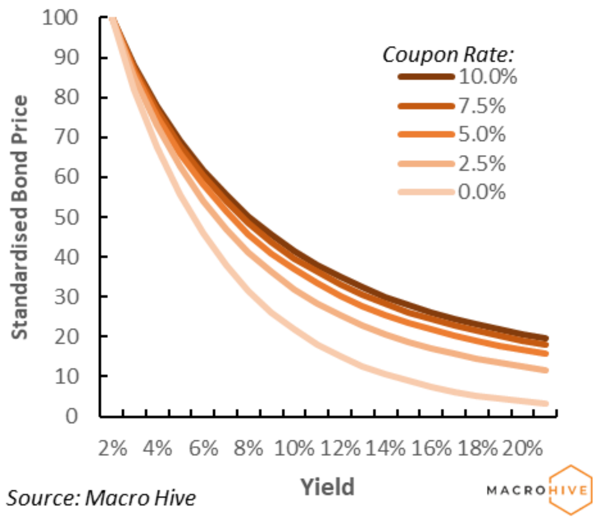
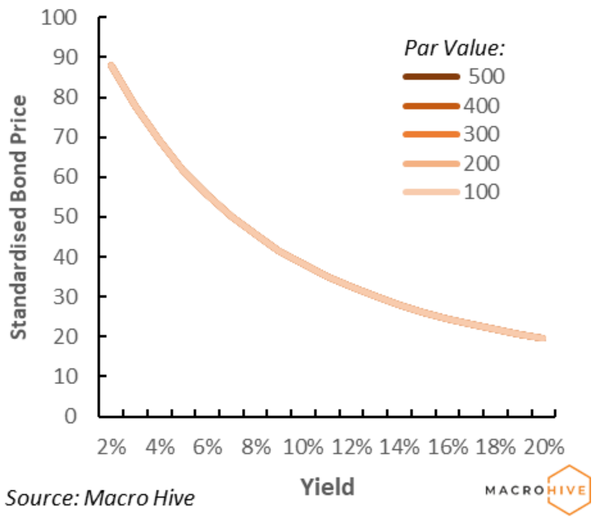


Chart 4: Convexity Does Not Change as Par Value Changes



Importantly, the Price-Yield curve is ‘curved’. This is related to the concept of convexity. The curvature increases as the maturity increases (Chart 2) and as the coupon rate decreases (Chart 3). However, it does not change as the par value changes (Chart 4).

Duration as a Measure of Risk

How will the price of a bond change as the market yield changes?

To answer this, we need to understand the concept of *duration*. Duration is a time measure of a bond's (or fixed income portfolio's) price sensitivity to interest rate changes. It measures how long it takes, in years, for an investor to be repaid the bond’s price by the bond’s total cashflows.

The simplest example of duration is for a zero-coupon bond. This type of bond does not pay any coupons; it only pays the principal and interest on maturity. Thus, for a 10-year zero-coupon bond, the cashflow will be outstanding for 10 years. This is equivalent to saying the duration is 10 years which, in the zero-coupon case, equals the *time to maturity*.

In the above example, an investor is said to be *fully exposed* to interest rate changes – any interest rate change will impact your entire cashflow. This is a special case, though. Time to maturity and duration are usually different. Most bonds pay regular coupons, which means part of the bond's value is banked over time, reducing the bond's exposure to interest rate changes.

To measure the drop in duration relative to the time to maturity, we simply need to account for these earlier coupon payments by weighting the cashflows and taking their present values (Table 2). Sticking with a 10-year bond, but with a coupon rate of 5% (accrued annually) and a market interest rate of 2%, the *Macaulay Duration* falls to 8.35 years (Table 2).

The fall in duration represents a *fall* in the bond risk – we have become less exposed to interest rate changes. In principle, duration is lower for (i) a shorter maturity date, (ii) a higher coupon rate, and (iii) a higher yield.

Table 2: Macaulay Duration and Time to Maturity

1	B	C	D	E	F	G
2			*Bond*			
3	Maturity (years)		10			
4	Principal		100			
5	Yield		2%			
6	Coupon Rate		5%			
7						
8					=C10/(1+C5)^B10	
9	Years	Cashflow	PV	PV*t		
10	1	5	4.90	4.90	=D10*B10	
11	2	5	4.81	9.61		
12	3	5	4.71	14.13		
13	4	5	4.62	18.48		
14	5	5	4.53	22.64		
15	6	5	4.44	26.64		
16	7	5	4.35	30.47	Note: coupon payments are in	
17	8	5	4.27	34.14	years. If they had been in half	
18	9	5	4.18	37.65	years, we would divide by 2	
19	10	105	86.14	861.37		
20						Bond price
21	Time to Maturity		10			
22	Duration (Macaulay)		8.35		=SUM(E10:E19)/SUM(D10:D19)	
23	Bond price ($)		126.95			
24	Modified Duration		8.19%		=(1/(1+C5))*D22	
25	Expected price ($)		137.34			

Duration and Expected Prices

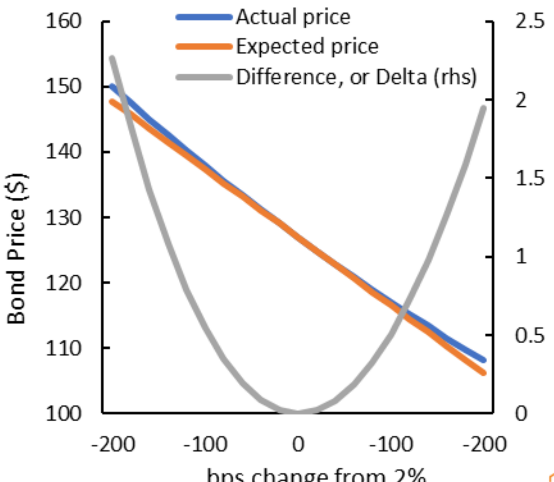
So far, we have only discussed how duration reflects interest rate risk. So how can we use it to calculate bond price changes? For this, we can use *modified duration*. Modified duration is a better measure of bond risk, or bond volatility, because it measures the expected change in a bond's price to a 1% change in interest rates.

In the above example, a 100bp fall in yields should lead to an 8.19% rise in the bond price. For a bond priced at $126.95, that is a $10.39 rise. So, the new estimated bond price is

$137.34. We can run the same exercise for different basis point changes and graph them alongside the actual prices (Chart 5).

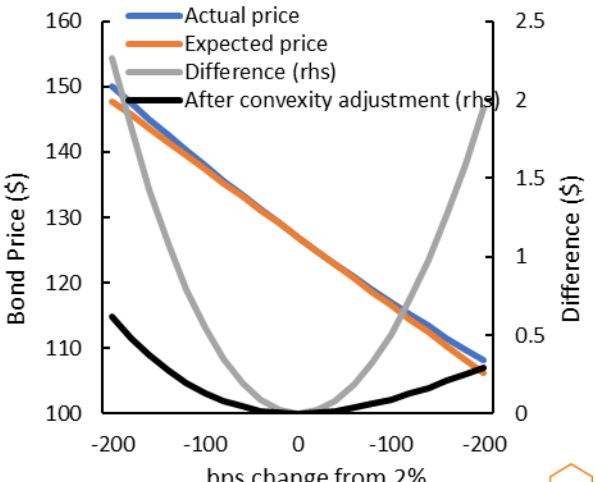
Two things emerge. First, the bond price predictions using duration are better for smaller changes in yields. In other words, the difference between the actual and predicted prices increases with larger yield changes. This has to do with *convexity*. Second, the duration prediction is symmetric – the predicted price adjustments for decreasing and increasing yields are the same.

Chart 5: Duration Is Good Approximation for Small Yield Changes



Source: *Macro Hive*

Chart 6: Adjusting for Convexity Yields far Better Price Predictions



Source: *Macro Hive*

Convexity

The difference between the actual price and the expected price in Chart 5 exists because the actual yield curve is 'curved', while duration is a linear measure (i.e., the first derivative). We can think of convexity as a second derivative. It takes into consideration the level of the yield as well as the accelerating rate of change in the yield.

Convexity is often presented in years (Table 3). In the same way we used duration to measure the approximate percentage change in a bond's price, we can do the same for convexity. Combining the two gives a far lower error between the actual and expected price (Chart 6). Note that it does not remove all the error though – a common misconception.

Table 3: Calculating Convexity

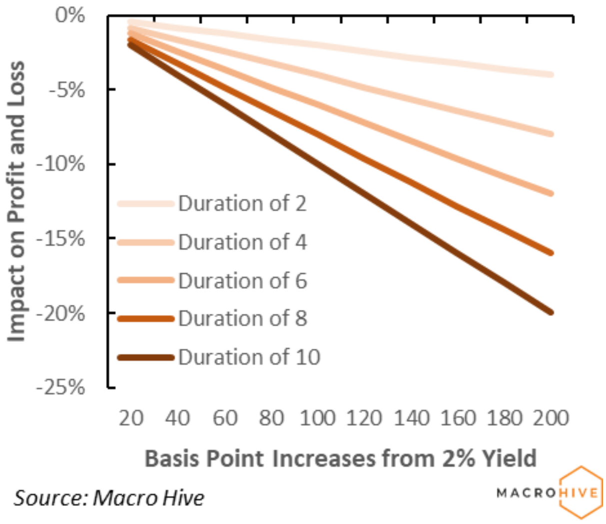
1	B	C	D	E	F	G	H
2			*Bond*				
3	Maturity (years)		10				
4	Principal		100				
5	Yield		2%				
6	Coupon Rate		5%				
7							
8							
9	Years	Cashflow	PV	PV*t(t+1)			
10	1	5	4.90	9.80	=D10*(B10*(B10+1))		
11	2	5	4.81	28.84			
12	3	5	4.71	56.54			
13	4	5	4.62	92.38			
14	5	5	4.53	135.86			
15	6	5	4.44	186.47			
16	7	5	4.35	243.76			
17	8	5	4.27	307.26			
18	9	5	4.18	376.54			
19	10	105	86.14	9475.02			
20	Sum		126.95	10912.47			
21							
22	Time to Maturity		10				
23	Convexity (years)		82.62		=E20/(((1+C5)^2)*D20))		
24							
25							

Duration and Profit and Loss

In practical terms, interest rate sensitivity is important for profit and loss (PnL). Typically, the lower the sensitivity/duration, the less changes in the interest rate will affect your PnL. As a rule of thumb, dividing duration by the change in yields will give an approximate impact on profitability.

In our example of a 10-year bond, duration was 8.19 (Table 2). If the yield rose 100bps to 3%, our rule of thumb gives us an 8% drop in the PnL. Graphically, we can show how the PnL changes with duration and the size of the basis point change (Chart 7). The larger the yield change and the higher the duration, the larger the losses. The losses are smaller when convexity is accounted for.

Chart 7: Duration affects Profit and Loss



Source: Macro Hive

Bottom Line

Duration and convexity are important concepts. Trying to understand them requires perseverance. Perhaps the best way to learn is using examples in Excel, and that is why we have provided some short examples. For a more rigorous understanding, we recommend 'Fixed Income Mathematics' by Robert Zipf. We also have a YouTube video explaining some of the concepts described above.